सक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2023

MTMACOR04T-MATHEMATICS (CC4)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Explain, with the help of uniqueness and existence theorem, that the differential equation

$$\frac{dy}{dx}=\frac{y}{x}$$

has infinite number of solutions passing through the point (0, 0).

(b) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of the differential equation

$$\frac{d^2y}{dx^2}-2\cdot\frac{dy}{dx}+2y=0$$

(c) Solve $(D^2-4D)y=x^2$, $(D\equiv\frac{d}{dx})$ by using the method of undetermined coefficients.

(d) Find the particular integral of the differential equation

$$(D^2-1)y=e^{-x},\ (D\equiv\frac{d}{dx})$$

(e) Locate and classify the singular points of the equation

$$x^3(x-2)\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+3xy=0$$

(f) Find the magnitude of the volume of the parallelopiped having the vectors $\vec{a}=-3\hat{i}+7\hat{j}+5\hat{k}$, $\vec{b}=5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{c}=7\hat{i}-5\hat{j}-3\hat{k}$ as the concurrent edges.

(g) If $\vec{F}=y\hat{i}-xz\hat{j}+x^2\hat{k}$ and C be the curve $x=t$, $y=2t^2$, $z=t^3$ from $t=0$ to $t=1$, then evaluate the integral $\int_C \vec{F}\times d\vec{r}$.

(h) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\hat{i}+\sin\omega t\hat{j}$, where ω is a constant. Show that the acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin.

2. (a) If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ and $\vec{c}\times\vec{a}=\vec{b}$, then show that $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular. 4

(b) Show that in general $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$; but if the equality holds, then either $\vec{b}$ is parallel to $(\vec{a}\times\vec{c})$ or $\vec{a}$ and $\vec{c}$ are collinear. 4

3. (a) Integrate the function $\vec{F}=x^2\hat{i}-xy\hat{j}$ from (0, 0) to (1, 1) along the parabola $y^2=x$. 4

(b) Prove that the necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is $\vec{a}\times\frac{d\vec{a}}{dt}=\vec{0}$. 4

4. (a) If $\vec{r}(t)=2\hat{i}-\hat{j}+2\hat{k}$ when $t=2$ and $\vec{r}(t)=4\hat{i}-2\hat{j}+3\hat{k}$ when $t=3$, then show that 4

$$\int_2^3\left(\vec{r}\cdot\frac{d\vec{r}}{dt}\right)dt=10$$

(b) Find the unit tangent, the curvature, the principal normal, the binormal and the torsion for the space curve 4

$$x=t-\frac{t^3}{3},\quad y=t^2,\quad z=t+\frac{t^3}{3}$$

5. (a) Solve $x^2\frac{d^2y}{dx^2}-3x\cdot\frac{dy}{dx}+y=\frac{\log_e x\sin\log_e x+1}{x}$. 4

(b) If y_1 and y_2 be two independent solutions of the linear homogeneous equation 4

$$\frac{d^2y}{dx^2}+P\cdot\frac{dy}{dx}+Q\cdot y=0$$

then show that the Wronskian $W(y_1, y_2)$ is given by

$$W(y_1, y_2)=A\cdot e^{-\int P\cdot dx},\text{ where } A \text{ is a constant.}$$

6. (a) Solve the equation 4

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x\sin x$$

by the method of undetermined coefficients.

(b) Solve 4

$$\frac{d^2y}{dx^2}+2\cdot\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$$

by the method of variation of parameters.

7. (a) Solve 4

$$\frac{d^2x}{dt^2}+\frac{dy}{dt}+x+y=t \quad , \quad \frac{dy}{dt}+2x+y=0$$

given that $x=y=0$ at $t=0$.

(b) Solve: $\frac{d^2y}{dx^2}+4y=4\tan 2x$ 4

8. (a) Solve the equation $\frac{d^2y}{dx^2}+(x-1)^2\frac{dy}{dx}-4(x-1)y=0$ in series about the point $x=1$. 5

(b) Show that the point of infinity is a regular singular point of the equation 3

$$x^2\cdot\frac{d^2y}{dx^2}+(3x-1)\frac{dy}{dx}+3y=0$$

9. (a) Solve: $(D^3-1)y=\cos^2\frac{x}{2}$ 4

(b) Solve $x^2\cdot\frac{d^2y}{dx^2}+x\cdot\frac{dy}{dx}-y=0$, given that $x+\frac{1}{x}$ is one integral. 4

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